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Dynamics of muonic atom cascade in hydrogen-helium mixtures

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Abstract. The deexcitation of excited muonic protium and deuterium in the mixture of hydrogen and helium isotopes is considered. Methods of experimental determination of the probability of direct atomic muon capture by hydrogen and muon transfer rates from excited muonic hydrogen to helium are proposed. Theoretical results for the population of the muonic atoms in the ground state, q_{1s}^{He}, are compared with the existing experimental data. Results obtained for $D_2 + ^{3,4}$He mixtures are of interest for investigation of nuclear fusion in $d\mu^{3,4}$He muonic molecules.

PACS. 36.10.Dr Positronium, muonium, muonic atoms and molecules

1 Introduction

Nuclear fusion in charge-asymmetric muonic molecules $h\mu Z$ ($h \equiv p, d, t$, and $Z \equiv {}^3\mathrm{He}, {}^4\mathrm{He}, {}^6\mathrm{Li}, {}^7\mathrm{Li}, {}^7\mathrm{Be}$...) provides a rare possibility of investigation the strong interaction at relatively low energies about keV. Results obtained may throw some light on the fundamental questions of physics (charge symmetry of strong interaction and isoinvariance, the character of P- and T-invariance) and may be also of interest for the problem of the primordial nucleosynthesis of light nuclei in the early Universe [1]. The collision energy region realized in muonic molecules is not accessible now in accelerator experiments due to small intensity of particle beams and small fusion cross-sections ($\sim 10^{-36} \div 10^{-42}$ cm^2) expected at these energies.

The intensive experimental research of nuclear fusion in deuterium-helium muonic molecule[1] is now carried out at meson factories [2–5]. However, for the correct interpretation of the experimental results obtained it is necessary to have detailed information about all processes occurring during the short time of deexcitation of the muonic atom and during the formation of the muonic molecule. These fast processes are: deexcitation of muonic hydrogen *via* radiative transition [6] and Auger transition [7–9], Stark mixing [7], Coulomb deexcitation [10,11], muon transfer from excited muonic hydrogen to other nuclei [12–16], and elastic collisions [17] responsible for the thermalization of the muonic hydrogen. Acceleration of muonic hydrogen during the cascade is mainly due to Auger transitions and Coulomb deexcitation.

The scheme of cascade for $d\mu$ atoms in D_2+He mixture is presented in Figure 1 as an example (the same scheme of deexcitation is supposed for muonic protium and tritium). Experimental time separation of all processes presented in Figure 1 is practically impossible due to their large number and short time of the cascade ($\leq 10^{-11}$ s at the liquid hydrogen density, LHD). However it is possible to separate the formation of the muonic hydrogen and the muon transfer from hydrogen to helium in excited states.

In this paper theoretical results for the probability of the deexcitation of $p\mu$ and $d\mu$ atoms to the ground state (the so called q_{1s}^{He} parameter)[2] in $H_2 + ^{3,4}$He and $D_2 + ^{3,4}$He mixture, respectively, are presented and compared with the experimental results for the $H_2 + ^4$He mixture. The probability for Coulomb muon capture[3] by hydrogen in different hydrogen-helium isotopic mixtures is also presented. The results for q_{1s}^{He} have been obtained using new theoretical data for muon transfer from excited states [16]. Auger transitions induced by collisions of muonic hydrogen with helium atoms are also used in the calculations.

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[1] Such a system is preferred now due to a lot of theoretical and experimental papers concerning the properties of this molecule.

[2] The index "He" was added to underline that hydrogen-helium isotopic mixtures are considered.

[3] It is called direct muon capture throughout the text.

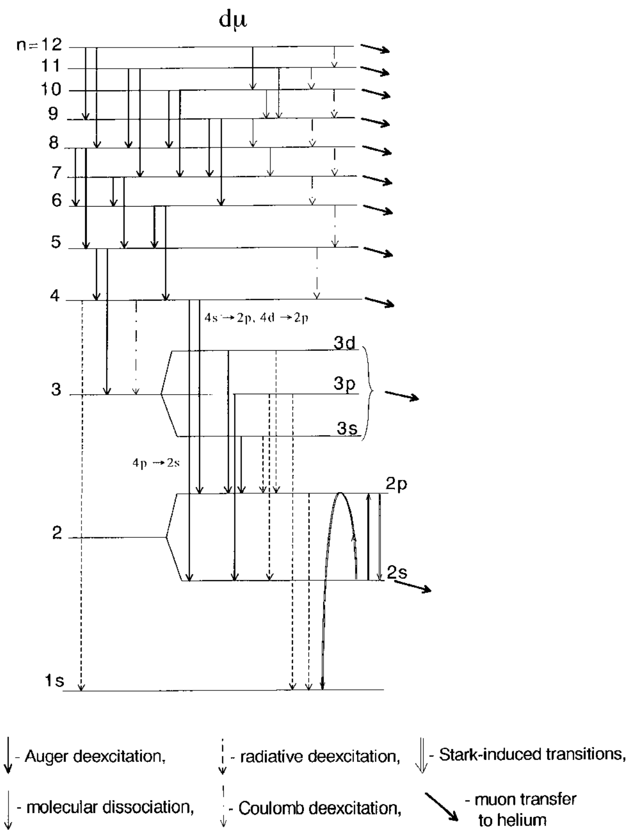


Fig. 1. Scheme of the muonic atom cascade for $d\mu$ atom in the $D_2 + He$ mixture. Only main radiative and Auger transitions are indicated.

2 Method of consideration

Since the properties of the nuclear fusion reaction in the $d\mu He$ molecules [18]

$$d\mu^3\mathrm{He} \xrightarrow{\lambda_f} \begin{cases} \alpha + p(14.7\ \mathrm{MeV}) + \mu \\ ^5\mathrm{Li} + \gamma(16.4\ \mathrm{MeV}) + \mu \end{cases} \quad (1)$$

$$d\mu^4\mathrm{He} \xrightarrow{\lambda_f} {}^6\mathrm{Li} + \gamma(1.48\ \mathrm{MeV}) + \mu \quad (2)$$

are usually studied by analysis of the yields and time distributions of reaction products normalized per $d\mu He$ complex formed it is necessary to find correctly the number of these complexes in the $D_2 + He$ mixtures.

In turn the number of these muonic molecules is proportional to the number of $d\mu$ atoms in the ground state which is determined by the probability

$$W = W_D\, q_{1s}^{\mathrm{He}} \quad (3)$$

that the muon stopped in the mixture will be captured by deuterium into an excited atomic orbit, and the resulting $d\mu$ atom will reach the ground state. W_D and q_{1s}^{He} are the corresponding probabilities. Probability W is a function of the mixture density, ϕ (usually expressed in the units of the liquid hydrogen density, LHD, $N_0 = 4.22 \times 10^{22}\ \mathrm{cm}^{-3}$), and of the helium concentration, C_{He}. It may be obtained from the analysis of experimental data by the method presented in [19]. However, experimental determination of W is very difficult because it should be measured together with the fusion rate, *i.e.* under the same experimental conditions. Therefore theoretical estimation of q_{1s}^{He} and W_D can be obtained if the scheme of deexcitation of muonic hydrogen and the corresponding reaction rates are well-known. The situation, however, is still unsatisfactory due to the lack of the cross-sections for deexcitation processes induced by collisions of muonic hydrogen with helium atom. Nevertheless the estimations of some missing data are proposed in this paper and results for q_{1s}^{He} are given.

Let us briefly describe the processes induced by muons entering a hydrogen-helium isotopic mixture.

Muonic hydrogen atoms are formed in highly excited states ($n \approx 8 \div 16$) and n-state distribution has a maximum for $n = 14 \approx \sqrt{m_\mu/m_e}$ [20] where m_μ and m_e are the reduced masses of the muonic and electronic hydrogen atom, respectively. The probability of direct muon capture by deuterium W_D can be expressed as

$$W_D = (1 + \frac{C_{\mathrm{He}}}{C_D} A)^{-1}, \quad (4)$$

where C_D is deuterium concentration ($C_{\mathrm{He}} + C_D = 1$) and A is the ratio of the capture rates for helium and deuterium: $A = 1.7 \pm 0.2$ [19]. Muonic hydrogen atoms undergo fast deexcitation to states with $n = 12$ due to dissociation of molecules and Auger transitions. So we suppose in our calculations that the cascade starts at $n = 12$.

For $n \geq 10$ deexcitation of muonic deuterium is dominated by dissociation of molecules in collision processes [7] (below $D_2 + He$ mixture is taken as an example)

$$(d\mu)_n + D_2 \to (d\mu)_{n'} + D + D, \quad (5)$$

where the transition energy matches the hydrogen molecule dissociation energy, $\varepsilon_{dis} \approx 4.7$ eV. Corresponding cross-sections have been approximated, (following [7]), by the geometrical ones.

For $4 \leq n \leq 10$ deexcitation due to ionization of hydrogen molecules [7]

$$(d\mu)_n + D_2 \to (d\mu)_{n'} + D_2^+ + e, \quad (6)$$

or helium atoms

$$(d\mu)_n + \mathrm{He} \to (d\mu)_{n'} + \mathrm{He}^+ + e \quad (7)$$

dominates (outer Auger effect), where transitions with $n - n' = 1$ are preferred[4]. Collisions with surrounding molecules and atoms lead to fast deexcitation to the states with $n \approx 4 \div 5$ during the time $\sim 10^{-12}\phi^{-1}$ s. For $n < 6$, radiative deexcitation dominates and the $1s$ state is reached in about 10^{-11} s. The corresponding radiative rates were obtained according to [6] and are presented in Table 1.

[4] The probability of Auger transitions is proportional to $(\Delta E)^{-1/2}$.

Table 1. Reaction rates for radiative deexcitation of the muonic deuterium atom and Auger transitions induced by collision of muonic deuterium with the deuterium molecule. Auger rates for deexcitation of muonic deuterium in collision with the helium atom are presented in parenthesis.

transition	Auger rates [10^{10} s^{-1}]	radiative rates [10^{10} s^{-1}]
2p → 1s	0.508 (1.017)	12.26
3p → 1s	0.075 (0.150)	3.273
3p → 2s	6.636 (13.44)	0.439
3d → 2p	19.11 (38.71)	1.265
3s → 2p	1.866 (3.780)	0.124
4p → 1s	0.025 (0.051)	1.334
4p → 2s	1.001 (2.020)	0.189
4d → 2p	2.135 (4.310)	0.404
4s → 2p	0.267 (0.539)	0.051
4 → 3	104.0 (215.7)	0.176
5p → 1s	0.012 (0.235)	0.673
5p → 2s	0.345 (0.695)	0.097
5d → 2p	0.656 (1.324)	0.184
5s → 2p	0.090 (0.181)	0.025
5 → 3	6.750 (13.84)	0.043
5 → 4	457.1 (989.9)	0.053
6p → 1s	0.006 (0.013)	0.386
6p → 2s	0.163 (0.329)	0.055
6d → 2p	0.294 (0.593)	0.101
6s → 2p	0.042 (0.085)	0.014
6 → 3	1.372 (2.802)	0.015
6 → 4	28.85 (60.73)	0.015
6 → 5	1437 (3327)	0.020
7p → 1s	0.004 (0.008)	0.242
7p → 2s	0.092 (0.185)	0.035
7d → 2p	0.160 (0.322)	0.061
7s → 2p	0.023 (0.047)	0.009
7 → 3	0.440 (0.898)	0.007
7 → 4	5.817 (12.13)	0.006
7 → 5	89.08 (195.2)	0.006
7 → 6	3620 (0.000)	0.009
8p → 1s	0.003 (0.005)	0.162
8p → 2s	0.057 (0.115)	0.024
8d → 2p	0.097 (0.196)	0.040
8s → 2p	0.014 (0.029)	0.006
8 → 3	0.181 (0.368)	0.003
8 → 4	1.871 (3.887)	0.003
8 → 5	17.79 (38.29)	0.003
8 → 6	223.2 (517.0)	0.003
8 → 7	0.000 (0.000)	0.004
9p → 1s	0.002 (0.004)	0.113
9p → 2s	0.038 (0.077)	0.017
9d → 2p	0.064 (0.129)	0.028

transition	Auger rates [10^{10} s^{-1}]	radiative rates [10^{10} s^{-1}]
9s → 2p	0.010 (0.019)	0.004
9 → 3	0.087 (0.176)	0.002
9 → 4	0.774 (1.604)	0.002
9 → 5	5.709 (12.18)	0.001
9 → 6	44.25 (99.42)	0.001
9 → 7	481.4 (0.000)	0.002
9 → 8	0.000 (0.000)	0.002
10p → 1s	0.001 (0.003)	0.082
10p → 2s	0.027 (0.054)	0.012
10d → 2p	0.045 (0.090)	0.020
10s → 2p	0.007 (0.014)	0.003
10 → 3	0.046 (0.094)	0.001
10 → 4	0.375 (0.775)	0.001
10 → 5	2.365 (5.020)	0.001
10 → 6	14.15 (31.30)	0.001
10 → 7	95.17 (225.6)	0.001
10 → 8	0.000 (0.000)	0.001
10 → 9	0.000 (0.000)	0.001
11p → 1s	0.001 (0.002)	0.062
11p → 2s	0.020 (0.040)	0.0001
11d → 2p	0.033 (0.065)	0.015
11s → 2p	0.005 (0.010)	0.002
11 → 3	0.027 (0.054)	0.001
11 → 4	0.202 (0.417)	0.001
11 → 5	1.150 (2.431)	0.0004
11 → 6	5.858 (12.84)	0.0004
11 → 7	30.35 (70.32)	0.0004
11 → 8	183.4 (0.000)	0.001
11 → 9	0.000 (0.000)	0.001
11 → 10	0.000 (0.000)	0.001
12p → 1s	0.001 (0.001)	0.048
12p → 2s	0.015 (0.030)	0.007
12d → 2p	0.024 (0.049)	0.011
12s → 2p	0.004 (0.008)	0.002
12 → 3	0.016 (0.033)	0.004
12 → 4	0.118 (0.243)	0.0003
12 → 5	0.623 (1.314)	0.0002
12 → 6	2.850 (6.213)	0.0003
12 → 7	12.55 (28.68)	0.0002
12 → 8	58.44 (0.000)	0.0003
12 → 9	0.000 (0.000)	0.0003
12 → 10	0.000 (0.000)	0.0003
12 → 11	0.000 (0.000)	0.001

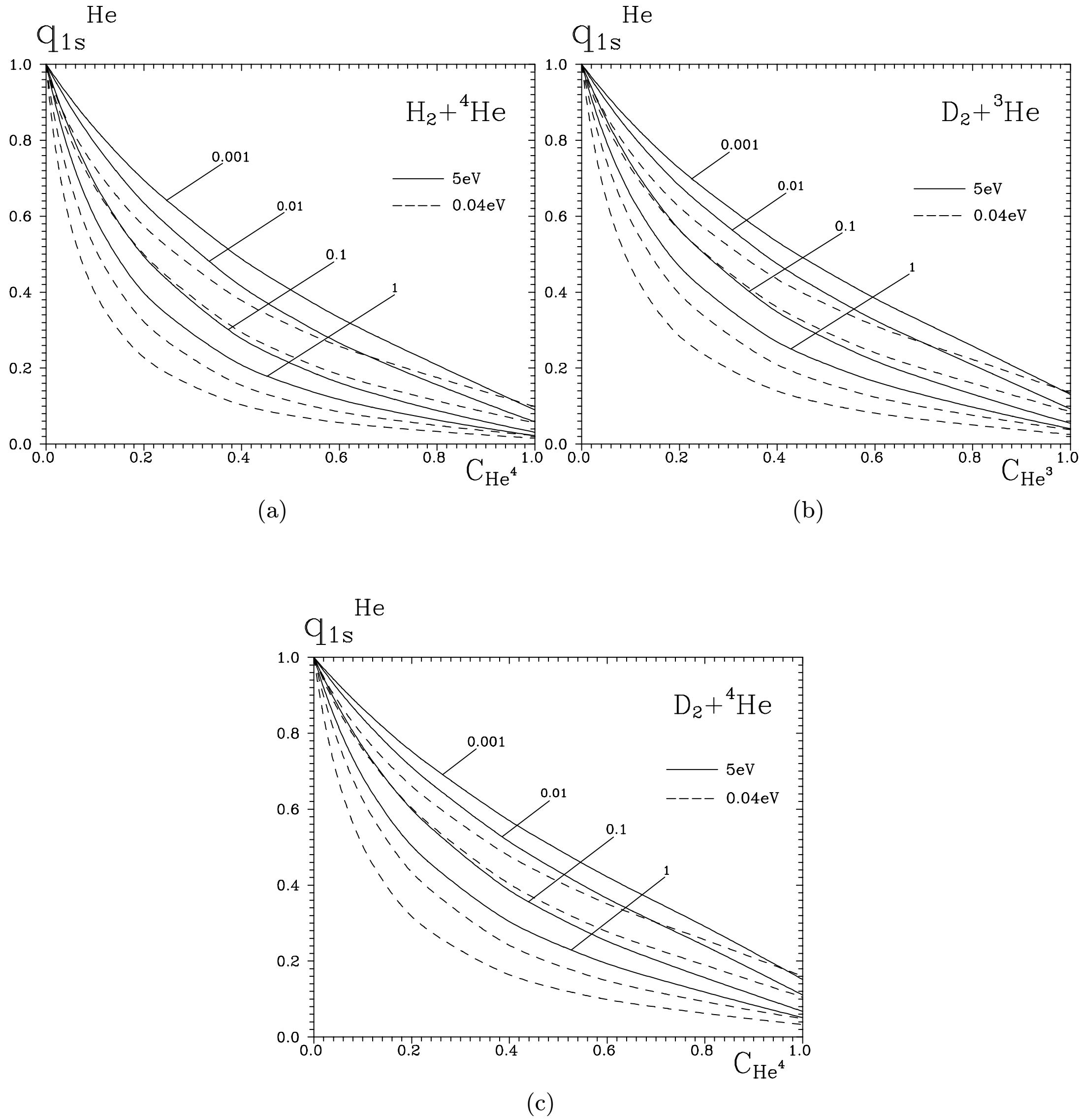


Fig. 2. Dependence of $q_{1s}^{\rm He}$ ($n \leq 12$) on $C_{^4\rm He}$ for $H_2 + {}^4He$ (a), $D_2 + {}^3He$ (b) and $D_2 + {}^4He$ (c) mixtures obtained for different mixture densities and different collision energies. The sequence of dashed lines is the same as solid ones.

Coulomb deexcitation [10,11]

$$(d\mu)_n + d \to (d\mu)_{n-1} + d \tag{8}$$

and

$$(d\mu)_n + \mathrm{He} \to (d\mu)_{n-1} + \mathrm{He} \tag{9}$$

also plays an important role.

For $n \geq 4$ the corresponding cross-sections for collision with deuterium nucleus are nearly comparable with those for molecule dissociation.

As it was shown in reference [21] about $6\% \div 15\%$ of muonic atoms deexcite *via* the $2s$ state from which the radiative transition to the $1s$ state is forbidden[5]. Therefore, collision induced $2s \to 2p$ transitions for collision energies, ε, greater than the Lamb-shift ($\Delta E_L \approx 0.2$ eV), and subsequent radiative $2p \to 1s$ transition become important in the cascade scheme [22]. For $\varepsilon < \Delta E_L$, $2s \to 2p \to 1s$ transitions due to Stark mixing of $2s$ and $2p$ states may occur [7].

There is no theoretical data for the above deexcitation processes induced by collisions with helium atom. The corresponding Auger rates were approximated by $2\lambda_A$, where λ_A were calculated according to formulae presented in [7] for helium ionization energy, $I_{\rm He} = 24.68$ eV, instead of the one for hydrogen molecule. They are presented in Table 1 (in parenthesis) for muonic deuterium together with Auger rates for the collision of muonic deuterium with the deuterium molecule. Reaction rates for Coulomb deexcitation and transitions between $2s$ and $2p$ (including

[5] The Lamb-shift in muonic hydrogen atoms is caused mainly by e^+e^- vacuum polarization which shifts the $2s$ level below $2p$.

induced $2s \to 2p \to 1s$ transitions) were supposed to be the same as those for the collisions of muonic deuterium with tritium and were taken from [10] and [11,22], respectively.

Deexcitation processes compete with muon transfer to helium nucleus

$$(d\mu)_n + \mathrm{He} \to (\mathrm{He}\mu)_n + d. \qquad (10)$$

The corresponding reaction rates were calculated in [16] for the principal quantum number $n \leq 5$. Because there are no theoretical results for the muon transfer to helium from $n > 5$, the corresponding transfer rates were supposed to be the same as the transfer rate for $n = 5$.

The quantity q_{1s}^{He} (ground state population of muonic hydrogen) depends on ϕ, collision energy ε, C_{He} and the competition between the deexcitation of muonic hydrogen and the muon transfer to He nuclei.

Theoretical results for q_{1s}^{He} calculated for $H_2 + {}^{3,4}\mathrm{He}$ and $D_2 + {}^{3,4}$ He mixtures presented in this paper were obtained in the so-called simple cascade model which assumes constant kinetic energy ($0.04 \div 5$ eV) of a muonic atom during the cascade. The method is based on the solution of a system of coupled linear differential equations corresponding to the scheme of the cascade presented in Figure 1 with the initial population of the states: $q_{12}^{\mathrm{He}} = 1$ and $q_{n<12}^{\mathrm{He}} = 0$. This model, applied in [11] for calculation of q_{1s} for $d\mu$ atoms in $D_2 + T_2$ mixture, gives results very close to those obtained by Monte-Carlo method that includes acceleration and thermalization of muonic atoms during the cascade [23].

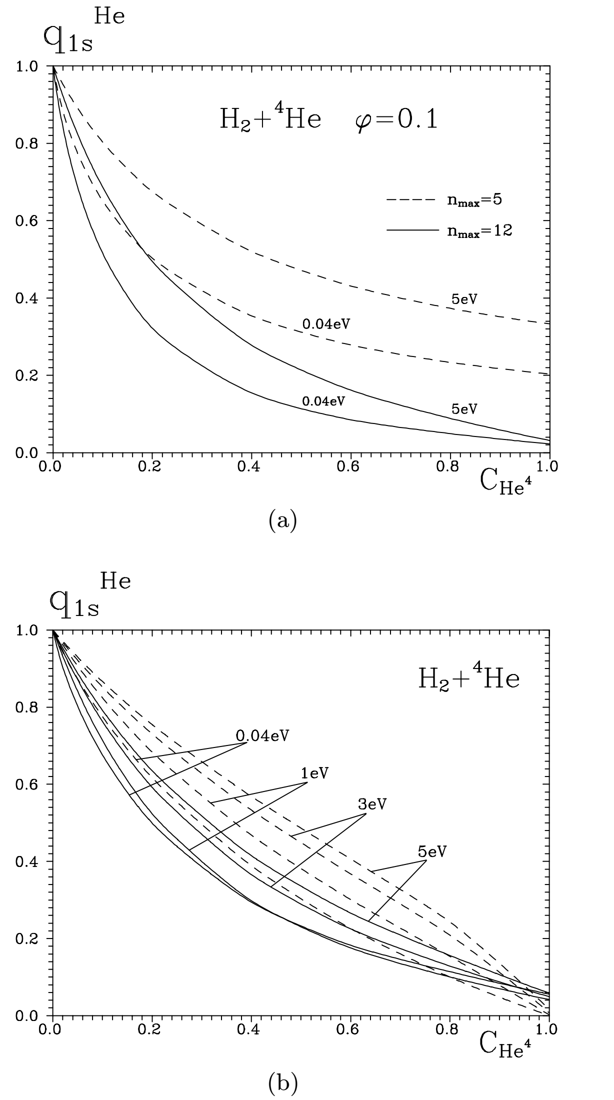


Fig. 3. Comparison of q_{1s} for the $H_2 + {}^4\mathrm{He}$ mixture for: (a) $\phi = 0.1$ and $C_{\mathrm{He}} = 0.4$ calculated for $n \leq 12$ and $n \leq 5$ at $\varepsilon = 0.04$ eV and 5 eV; (b) $n \leq 12$ and $\phi = 0.01$. Results were obtained for transitions presented in Table 1 and Figure 1 (solid lines) and only in Figure 1 (dotted lines). Collision energies, ε, are indicated at the curves.

3 Results

Figures 2a–c show the calculated dependence of q_{1s}^{He} (for $H_2 + {}^4$ He, $D_2 + {}^3$ He and $D_2 + {}^4$ He mixtures, respectively) upon the relative helium concentration C_{He} for different values of the mixture density ϕ and collision energies $\varepsilon = 0.04$ eV (dashed lines) and 5 eV (solid lines). All radiative and Auger transition rates presented in Table 1 have been included. Figure 3a obtained for $\varepsilon = 0.04$ eV and 5 eV illustrates the influence of muon transfer from $n > 5$. As follows from Figure 3b the contribution of secondary Auger and radiative transitions (presented in Tab. 1 but not presented in Fig. 1) is very important especially at large helium concentrations. It is due to the fact that Auger transitions, $n \to n-k, k = 1, 2, 3$, induced by collisions of muonic hydrogen with helium atoms are energetically forbidden (see Tab. 1).

Figure 4 shows the energy dependence of q_{1s}^{He} for $\phi = 0.1$ and $C_{\mathrm{He}} = 0.4$ for all mixtures considered. The character of the energy dependence is not sensitive to these parameters. It should be noted, however, that unlike the case of the deuterium-tritium mixture [11,12] the energy dependence of q_{1s}^{He} for deuterium-helium mixture is rather weak. The analogous comparison of q_{1s}^{He} in the $H_2 + {}^4\mathrm{He}$ mixture with that in the H_2+D_2 mixtures was given in [24] As was already indicated in [14], the weak energy dependence of q_{1s}^{He} is due to a relatively small rate of the muon transfer to helium from the metastable $2s$-state [13,15,16]. It is about an order of magnitude smaller than the rate for muon transfer to tritium [11]. At the same time the reaction rate for deexcitation of the $2s$-state (due to $2s \to 2p$ and the subsequent $2p \to 1s$ transition) has a strong energy dependence due to the presence of the $2s - 2p$ Lamb-shift threshold, ΔE_L. Therefore, the increase of q_{1s}^{He} for $\varepsilon > 0.2$ eV is much less pronounced than the increase of q_{1s} in the $D_2 + T_2$-mixture.

The decrease of q_{1s}^{He} for $\varepsilon < 0.2$ eV (see Fig. 4) is mainly due to the rise of muon transfer rates in this energy region [16]. Additionally, increasing with energy the $2p \to 2s$ transition rate [22] enhances the population of $2s$ state. It results in faster muon transfer to helium from

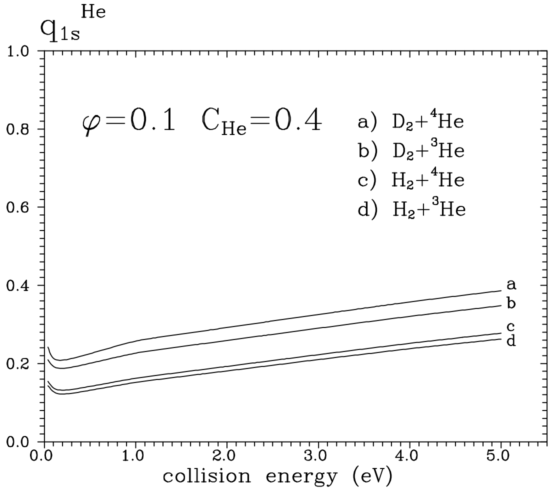
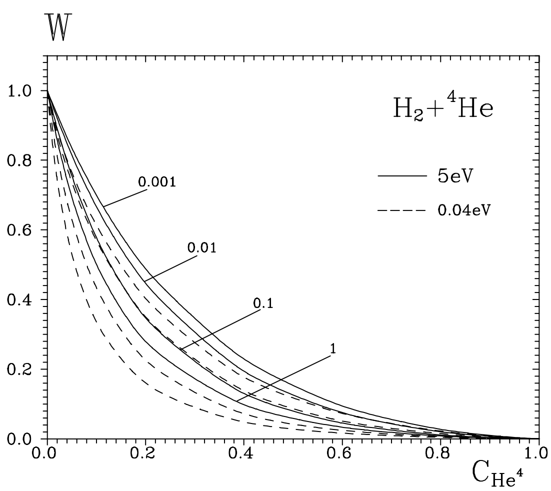
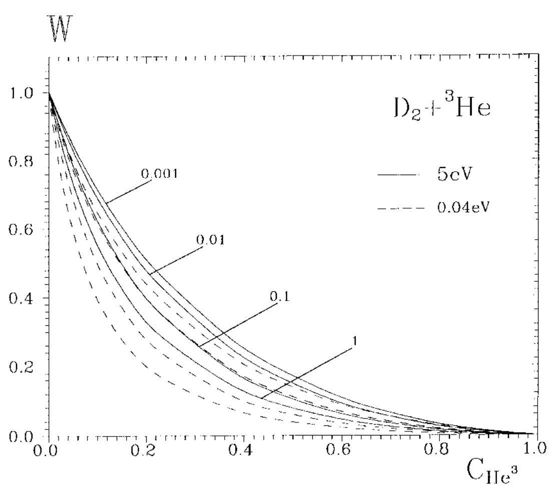


Fig. 4. Energy dependence of q_{1s} obtained for $\phi = 0.1$, $C_{\rm He} = 0.4$ and different isotopic mixtures.



(a)



(b)

Fig. 5. Dependence of W on $C_{\rm He}$ for H_2+^4He (a) and D_2+^3He (b) mixture for ϕ and ε indicated at the curves.

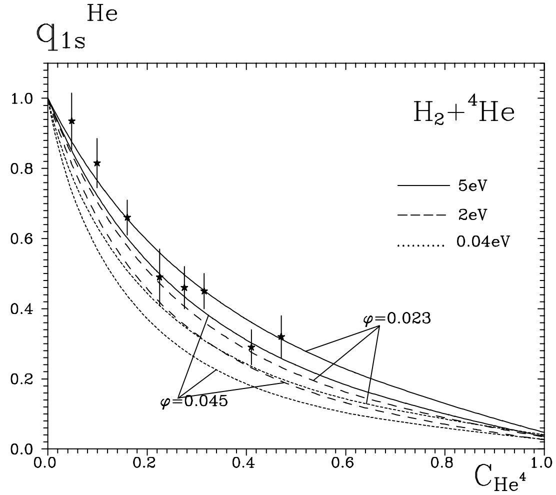


Fig. 6. Comparison of the calculated $q_{1s}^{\rm He}$ for H_2+^4He mixture with the experimental data.

Table 2. Comparison of the experimental and calculated values of W and $q_{1s}^{\rm He}$ for H_2+^4He at different ϕ and $C_{\rm He}$. Experimental errors are indicated in parenthesis. Theoretical data were calculated for $\varepsilon = 0.04$ eV and 5 eV (in square brackets).

	experiment (H_2+^4He)				*theory* (H_2+^4He)	
	ϕ	$C_{^4He}$	$q_{1s}^{^4He}$	W	$q_{1s}^{^4He}$	W
1	0.031	0.048	0.94(8)	0.87(3)	0.77 [0.87]	0.71 [0.80]
2	0.032	0.099	0.82(7)	0.70(4)	0.61 [0.75]	0.52 [0.63]
3	0.023	0.160	0.66(5)	0.50(2)	0.51 [0.66]	0.38 [0.50]
4	0.038	0.225	0.49(8)	0.33(5)	0.34 [0.53]	0.25 [0.35]
5	0.027	0.275	0.46(6)	0.28(3)	0.34 [0.49]	0.21 [0.30]
6	0.035	0.315	0.45(5)	0.25(2)	0.28 [0.42]	0.16 [0.24]
7	0.033	0.410	0.29(5)	0.13(2)	0.21 [0.34]	0.10 [0.16]
8	0.045	0.470	0.32(6)	0.12(2)	0.16 [0.27]	0.06 [0.11]

this state and additional decrease of $q_{1s}^{\rm He}$. For the collision energy $\varepsilon > 0.2$ eV the $2s \to 2p$ transition is switched on and the $2s$ state deexcites to the ground state due to the $2s \to 2p$ transition and the subsequent $2p \to 1s$ radiative transition. The increase in the $2s \to 2p$ transition rate with increasing energy and the decrease in muon transfer rates for $\varepsilon > 0.5$ eV lead to monotonical increase of $q_{1s}^{\rm He}$.

The values of $q_{1s}^{\rm He}$ for the $H_2+{}^4$He mixture in energy range $0 \div 1$ eV (see Fig. 2a) are some smaller than those presented in Figure 2 of [14]. It is because the cascade and transfer processes have been considered in the present paper for $n \leq 12$ (see Fig. 1 and Tab. 1) whereas in [14] they were considered for $n \leq 5$.

The dependence of W on $C_{\rm He}$ for different ϕ and $\varepsilon = 0.04$ eV and 5 eV for $H_2+{}^4$He and $D_2+{}^3$He mixtures is shown in Figures 5a and 5b, respectively. Some of these results are also presented in Table 2 together with the calculated values of $q_{1s}^{\rm He}$ and W and experimental ones [19,25] obtained for $H_2+{}^4$He mixture.

Figure 6 presents the experimental values of $q_{1s}^{^4\rm He}$ [19,25] and the theoretical ones calculated for the same target densities and for the collision energies ε indicated at the curves. As is seen, the agreement between the experimental and theoretical data is obtained for collision energy $\varepsilon > 0.04$ eV. As follows from this, the average energy of muonic hydrogen atoms, corresponding to their real energy distribution in excited states with $n \approx 2 \div 4$, is much greater than the thermal energy. This results in greater probability of the Stark $2s \rightarrow 2p$ and induced $2s \rightarrow 2p \rightarrow 1s$ transitions.

One can conclude that agreement between the experimental and theoretical data is possible for the indicated $H_2 +^4$ He mixture if the $q_{1s}^{\rm He}$ values are calculated for a high collision energy $\varepsilon \sim 2 \div 5$ eV of excited muonic hydrogen and He atom (see Tab. 2 and Fig. 6). The recent q_{1s} experimental data for $H_2 + D_2$ mixture indicate also on high $\varepsilon \sim 5$ eV [26,27].

In the conclusion, we can argue that comparison of the experimental data of W (and correspondently $q_{1s}^{\rm He}$) obtained for different ϕ and $C_{\rm He}$ with the corresponding theoretical ones could allow one to verify the cascade scheme and to obtain transfer rates from excited muonic hydrogen to helium using the χ^2 analysis.

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